# THIRD-ORDER VIABILITY IN RADICAL CONSTRUCTIVISM

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*In this paper, we will address the methodological problem of extending second-order models in radical constructivism. As a solution, we propose to convert second-order models to third-order viable first-order models. This conversion consists of identifying what information the students could not precisely access, in the case that their behaviors were the most rational in the situation. Because of this conversion, any converted model is expected to be viable, not only for the observer (first-order viable) and for the observed subject (second-order viable), but also for other persons (third-order viable). We will discuss the educational implications.*

## INTRODUCTION

Radical constructivism (RC) is a philosophy of knowing which assumes:

> [1-a] Knowledge is not passively received, either through the senses, or by way of communication; [1-b] knowledge is actively built up by the cognizing subject. [2-a] The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability; [2-b] cognition serves the subject's organization of the experiential world, not the discovery of an objective ontological reality. (von Glasersfeld, 1995a, p. 51; Numbering added for citation)

One of the recent contributions of RC to mathematics education is the study of how *second-order models* are developed, and what potential impact RC may have on practice (Ulrich, Tillema, Hackenberg, & Norton, 2014). A second-order model is a model of a particular student's thinking processes, used to explain the observer's experience (Steffe & Thompson, 2000, p. 205). The reflective use of second-order models can provide strong guidance for teachers and researchers (Thompson, 2000, pp. 303–304). However, according to Sánchez Gómez's (2014) comment on Ulrich et al. (2014), the validity of the extension of second-order models for a particular student to new students is not methodologically warranted.

Although Tillema, Hackenberg, Ulrich, and Norton (2014) claimed that Sánchez Gómez's interpretation was "different from [Tillema et al.'s] understanding of the purpose of creating second-order models and the nature of these models" (p. 355), this does not seem to be a valid counterargument against Sánchez Gómez (2014) from the RC perspective itself. Following the RC principle [2-a] cited above, any interpretation should be viable for the interpreter. RC should not be able to claim that Sánchez Gómez misinterprets. In this paper, we will address the methodological problem of extending second-order models in RC (the extension problem). For this, we will start with a review of the nature of knowing in RC.

## NATURE OF KNOWING IN RC

The concept of *viability* is the most important concept in this paper. For students, the

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condition that *pieces of knowledge reflect the absolute truth* is neither necessary nor sufficient for their use. Rather, students seem to use them if they are *viable*; that is, "if they fit the purposive or descriptive contexts in which [the students] use them" (von Glasersfeld, 1995a, p. 14). RC is "uninhibitedly instrumentalist" (p. 22). The term *viable* is not a synonym for the terms *true* or *valid*. The observation that a particular piece of knowledge is viable for the subject does not mean that the person has a particular justified belief. Rather, it means that in a particular situation, the subject has the *disposition* to *make a decision* to use a particular cognitive tool.

This instrumentalist view of knowledge becomes more clear with the concept of *knowledge-how* in the sense of Ryle (1949). For example, the reason that one can speak logically is not that one can recall the rules of inference and apply them, it is because one implicitly knows how to speak in such a way. Such implicit knowledge is described as *knowledge-how*, while a propositional knowledge is described as *knowledge-that*. With this terminology, we can say that RC does not acknowledge any piece of *knowledge-that* because we cannot have access to the absolute truth. Rather, RC only acknowledges *knowledge-how*, and regards any type of knowledge (e.g., ideas, strategies, cognitive structures, or models) as *knowledge-how*.

It is noteworthy that even cognition like "seeing ... as ..." or "recognizing ... as ..." is treated as *knowledge-how*. For example, suppose that a subject uses a stone to drive a nail into a wall because s/he cannot immediately access a hammer (cf. von Glasersfeld, 1995b, p. 374). Let *S* be the subject. Seeing a stone as a hammer is *S*'s *knowledge-how*. The reason that *S* saw the stone as a hammer is not that *S* volitionally decided to see a stone as a hammer, and so actually saw the stone as a hammer. It is because *S* implicitly knew how to see the stone as a hammer, for example, how to decide which parts of the stone would correspond to the face, or the grip of a hammer. If *S* knew only how to see a small and hard substance as a stone, the stone would be only a stone for *S*.

*S* cannot arbitrarily construct any *knowledge-how* which *S* wants, because the environment constrains the viability of *S*'s *knowledge-how* (von Glasersfeld, 1990, p. 24). However, note that *S* can arbitrarily construct any *knowledge-how* as long as the constraints are not violated. Whatever *S* learns from the fact, is what *S* selectively and hypothetically constructs. In the above example, the expectation that the stone can be used as a hammer is an ill-grounded hypothetical construct. Generally speaking, *S* actively uses, not only justified knowledge, but also hypothetical knowledge when trying to achieve a particular goal. In this paper, we will call this characteristic of knowledge use as *the hypothetical nature*.

In summary, any *knowledge-how* construction and use are valid for *S*. In this sense, even a young, uneducated child is regarded as a mini scientist (or a mini mathematician). This view has shed light on the nature of children's construction of knowledge. However, it diminishes the distinction between naïve, and sophisticated, knowledge construction. Especially within the context of second-order models, any methodological critique of the use of second-order models becomes invalid, because any temporal knowledge construction is scientifically valid. In the next section, we will address this problem.

## REAL PROBLEMS IN EXTENDING SECOND-ORDER MODELS

The purpose of building a second-order model is "to organize his or her experience in a way that helps him or her effectively interact with multiple students at different stages of reasoning, often at the same time" (Tillema et al., 2014, p. 356). That is, building and using second-order models is the observer's *knowledge-how*.

Let us take an example of extending second-order models from Ulrich et al. (2014). They used the models of *two composite units* (two units of units) and *only single composite unit* (one unit of units) to explain responses from a sixth grade student (Charice). Two problems were given to her for promoting a meaning of powers.

> *The Two-Suit Card Problem*: You have the Ace through King of hearts (13 cards). Your friend has the ace through King of spades (13 cards). You and your friend make two-card hands by drawing a card from your hand, then drawing a card from your friend's hand, and putting them together. Use an array to show how many different two-card hands you could make.
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> *The Password Problem*: students are creating two-character passwords for their computer account at school (e.g., "FD" is an example password). They can choose from the characters A through N to create the password. How many two-character passwords are possible (Assume "FD" and "DF" count as different passwords)? (p. 333)

The teacher/researcher expected Charice to solve each problem with two composite units. The two sets of 13 hearts and 13 spades are regarded as two units of units, because we must choose one from each of them in the Two-Suit Card Problem. The two sets of 14 characters are regarded as two units of units because we must choose one from each of them in the Password Problem. For the first problem, the teacher gave Charice all of the hearts in a deck of cards, and for the second problem, the teacher presented Charice with 14 cards on which one of the letters A through N was printed. Although Charice easily solved the first problem, she could not solve the second problem, and expressed that there is no number that is multiplied by 14. Because she seems to make passwords by choosing from a single set of 14 characters, her thinking is constrained by the model of only single composite unit (pp. 333–334).

This extension of the second-order model is valid due to the hypothetical nature of knowledge use. It is, in fact, hypothetical, but reasonable and promising. Although Tillema et al. (2014) claimed that Sánchez Gómez's (2014) interpretation was different from theirs, we can now properly understand both Sánchez Gómez's and Tillema et al.'s interpretations. The former viewed the hypothetical nature of the extension as a methodological problem, while the latter accepted the risk of the potentially invalid extension for possible future benefit.

"A drowning man will clutch at a straw." That is, when a person must make a decision without enough justified knowledge, s/he tends to use any knowledge, even ill-grounded knowledge, in order to make the decision. Using ill-grounded knowledge and risking biased decisions is not always irrational, because making no decision and taking no action may make the situation worse than making the wrong decision. In the case of extending the second-order model, as cited above, the purpose is to promote effective interactions with students. No matter how likely the extension of a model is to be invalid, it is more rational for teachers to extend it, and to interact with their students, than to make no decision and take no action.

Even if so, we cannot say that any methodological critique of extending second-order models is meaningless. Any extension of second-order models is idiosyncratically rational and valid for the extender himself or herself, while it is not always viable for others. Thus, as a methodological critique, we can ask the following question: *How likely is the second-order model to be second-order viable?* In RC, *first-order viability* is the viability of a piece of knowledge for the knowledge holder, while *second-order viability* is the viability of the piece of knowledge "not only in [the knowledge holder's] own sphere of actions but also in that of the other" (von Glasersfeld, 1995a, p. 120). Simply speaking, we can say that Ulrich et al.'s (2014) extension was not viable for Sánchez Gómez. This does not necessarily mean that he misunderstood Ulrich et al.'s intention to promote effective interactions with students. Rather, it means that he did not think that he would extend and use the second-order model in the same way if he were the teacher of Charice. For example, Ulrich et al. (2014) wrote that the teacher/researcher

> … asked [Charice] to elaborate on her observation, which opened the way for her to continue thinking about a solution to the problem. As she moved forward in her solution, she determined that she could pair A with each of the 13 other letters, then concluded that A could also be paired with itself so that A could be paired with 14 letters, and eventually that each of the 14 letters could be paired with 14 other letters. (p. 335)

The above quotation expresses only what decision the teacher actually made. It does not include the information on why she determined to teach in such a way. It is implicit from the reader's point of view how the extended second-order model works when the teacher made the decision. The proverb "a drowning man will clutch at a straw" is second-order viable because we share the implicit assumption that there is nothing but the straw around the man. We naturally think that we would clutch at a straw if we were drowning. On the other hand, the second-order model of only one composite unit does not necessarily have high second-order viability because we cannot assume that there are no different second-order models. Thus, in the next section, we will discuss how we can make the model to be second-order viable.

## FROM SECOND-ORDER VIABILITY TO THIRD-ORDER VIABILITY

A possible reason that the second-order model of only one composite unit does not have high second-order viability is that it does not explain why some students think in such a way. Any second-order model is problematic for the same reason.

This problem is similar to Confrey's (1991) critique of using the label *misconception*:

> Labeling a student's model as a *misconception* fails to take in consideration the perspective of the student, for whom the belief may explain all instances under consideration, and fail only in cases to which s/he is not privy. [...] Finally, others have chosen more simply *conception*, which omits any indication that the perspective may deviate considerably from the expert's position. (p. 121)

Although the term *second-order model* does not have a modifier like *mis*, labeling a student's thinking as a second-order model is often equal to *misconception*. The second-order model has provided the distinction between correct and incorrect thinking. It has not provided the explanation of idiosyncratic rationality for students. Unless we identify the students' idiosyncratic reason that they think with only single composite unit, we still implicitly keep the label *mis*.

For RC, it is important to explain idiosyncratic rationality. Based on the RC principle [2-b], all decisions are idiosyncratically rational. The fact that a person made a particular decision means that there was at least a moment when s/he thought that it was the most rational decision, even if it is later understood to be irrational, based on new information. Since human beings have only a limited capacity to deal with incoming information, we cannot deal with too much information at one time. We become, however, able to deal with a great deal of data at once if we acquire the ability to abstract and mathematise information. Thus, in mathematics education, we should assume that novices might not know what information is important to them, while focusing on that which is trivial; but the novices will always behave in the most rational way *from their own point of view*. Lacking the knowledge of what information is important is not necessarily careless; it is a result of overconcentration on other pieces of information. This characteristic of novices is referred to as *local rationality*. In contrast, experts' rationality, developed by dealing regularly with relatively large amounts of information, is referred to as *global rationality*.

Although the use of second-order models fails to explain the local rationality of students, there is one possible solution to this problem. It is to convert the already existing second-order models to *the observer's first-order models*. This would be achieved by identifying the information the students were not able to access, provided their behavior was otherwise rational, given the information they *did* have.

For example, in case of Charice, the teacher (i) presented the Password Problem to Charice, (ii) demonstrated a way of creating two-letter passwords with a set of 14 cards, and (iii) asked Charice if she could make a chart to solve for the total number of passwords. Then, (iv) Charice wrote down the list of 14 characters, and stopped solving (p. 333). In this case, Charice's response would be considered rational, even from our perspective, if step (i) did not exist. The teacher's question at step (iii) seems to shift Charice's interest from the Password Problem to the question itself. At this moment, she lost the need to solve the Password Problem, and suddenly needed to make a chart. According to the assumption of local rationality, Charice probably over concentrated on creating a chart. This situation is one in which the information presented at steps (i) and (ii) became inaccessible.

Suppose that we were she, and that we could not access the precise information presented at steps (i) and (ii). Then, for making a chart, we would have to recall how we had made similar charts. Although we had made them by choosing two units (e.g., hearts and spades in the Two-Suit Card Problem) until now, we could find only one unit (a set of 14 cards). We would not notice that we used only one set twice previously, because the information that we used a set twice was inaccessible because of our current assumption. As a result, we would be confused, because we could not make a chart in the same way as before. In this way, we find that we ourselves would also use only one composite unit, if important information suddenly became inaccessible.

There are two advantages to the above conversion. First, the converted model enables the teacher to empathise with the students. The model of only one composite unit is converted from a second-order model for explaining the students' behavior, to a first-order model for explaining the observer's virtual experience. While second-order models are only first-order viable, the converted models are not only first-order viable, but also second-order viable for the observer, because it is viable not only for the observer, but also for the students. Because of this second-order viability, it is easier for the observer to understand the students' thinking with the converted models, than with the corresponding second-order models.

If we understand the local rationality of the students, the question of why there are such students is easily answered. The reason that there are students modeled by the model, is that some teachers' behavior unintentionally causes them to lose focus on the important information. For example, in case of Charice, the reason that she used only one composite unit is that the teacher's question at the step (iii) unintentionally caused her to lose the focus on the information in steps (i) and (ii). Although, of course, there is the possibility that the student is careless, attributing the cause of the student's behavior to the teacher's behavior makes it easy for the teacher to empathise with the student, and to consider what to do next.

The second advantage is that the converted model is expected to be not only second-order viable for the observer, but also second-order viable for the third person, like the readers of research papers. For example, although the second-order model of only one composite model does not seem to be viable for Sánchez Gómez, the converted model is viable, even for him, because it provides him with a method to empathise with the student. If it is still not viable for him, the reason is not that the converted model itself lacks viability, but that he cannot accept the assumption of local rationality. The conversion includes the process of explaining novices' local rationality so that even experts can understand it. Thus, as long as the nature of local rationality is assumed, any converted model is expected to be viable not only for the observer (i. e., the first person "I") and for the observed subject (i. e., the second person "you"), but also for other persons (i.e., the third persons "they"; e.g., the readers of the research papers). Second-order viability is stronger than first-order, and this new viability is stronger than second-order. Therefore, we will call it *third-order* *viability*.

Third-order viability is the key concept for solving the extension problem. The first-order viable second-order models are retrospectively built after some observation. Since it strictly depends upon observation, the models are fragile. Since they are not related to any other information, we are constrained to use them without any supplemental information. On the other hand, the third-order viable first-order models are assimilated into the observer's existing knowledge when they are converted from the corresponding second-order models. That is, much of the observer's past experience will support using the third-order viable first-order models. Although it is never safe, in the sense that they are only approximate models of absolute reality, it is useful in that the observer can use them in accordance with his or her own empirically, well-tested, viable, existing knowledge. In the next section, we will discuss how to use third-order viable first-order models as educational tools.

## EDUCATIONAL IMPLICATIONS AND CONCLUSION

Before discussing the implications of using third-order viable first-order models for education, note that we do not intend to criticise Charice's teacher in the discussion below. According to the assumption of local rationality, we believe that the teacher's real-time practice was done to the best of her ability and understanding, based on her experience. We do not believe that the teacher should have done anything differently. Here, we will discuss what we could do in similar circumstances as Charice's teacher.

Even if we discover the cause of the students' behavior, we must keep in mind that eliminating that cause is not always the best way to improve the lesson. For example, the cause of Charice's behavior seemed to be the teacher's question as to whether Charice could make a chart. However, if the teacher presented only the Password Problem itself, and provided no support to solve it, then Charice could not know what to do. Since ancient times, it has been well known that introducing sub-questions in assignments is one of the most effective ways of supporting students. To cease introducing sub-questions would be ineffective.

Let us elucidate the model of Charice's thinking: From the hypothesis that she lost the need to solve the Password Problem because of the requirement to make a chart, it is deduced that she was not ready to make a chart. In fact, no one can *a priori* determine what a given problem will require one to do. It is determined after solving the problem. Thus, generally speaking, a student needs to notice, by himself or herself, that making a chart is a useful solution for this problem.

Keeping in mind the above, we can provide a useful approach to teaching the Password Problem in the future. A possible situation in which a student notices the usefulness of creating a chart is one in which s/he must make new passwords one after another. For example, suppose that (i) students engage in a game; (ii) it requires them to make new passwords by turns; and (iii) one wins the game by making more passwords than the other students make. In the game, the students may randomly create passwords, but gradually they will realise that it becomes more difficult to create new passwords according to the rules the longer the game lasts. Then, they will realise that a system for generating these passwords is required. The need to make a chart will arise. If they make many passwords by themselves, we can also expect them to notice on their own that the pile should be used twice. In this case, the role of the teacher will not include prompting them to make a chart. Rather, the teacher would (i) find the first student who makes a chart, (ii) share the information that that particular student is creating and using a chart, and (iii) encourage the students to consider what kind of chart would be the best for winning the game. This approach would be expected to help the students to understand the usefulness of tables as a preliminary step towards understanding powers.

In this paper, as a solution of the methodological problem of extending second-order models, we proposed to convert second-order models to third-order viable first-order models. However, the paper does not provide a general strategy for converting second-order models. The method of conversion still depends on each second-order model. Developing a practical strategy is an issue to be addressed in the future.

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